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SEMIRINGS OF CONTINUOUS PARTIAL NUMERICAL FUNCTIONS WITH EXTENDED ADDITION

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The article deals with the semiring of all continuous functions on a topological space X with values in the topological field of real numbers $\mathbb{R} \cup \{\varnothing\}$, which is completed by the isolated zero $\varnothing$. Operations of addition and multiplication over functions are pointwise. This semiring coincides with the semiring $CP(X)$ of all continuous partial real-valued functions whose domains are clopen subsets of the topological space X. The maximal ideals and maximal congruences of the semirings $CP(X)$ are described. A class of maximal subalgebras in the semirings $CP(X)$ is found. It is proved that any Hewitt space X is defined by the semiring $CP(X)$. The case of a finite discrete space X is studied.

Keywords: extended field of real numbers, topological space, semiring of continuous functions, partial function, ideal, congruence, subalgebra, definability.

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