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## EXISTENCE AND UNIQUENESS THEOREMS FOR ONE SYSTEM OF INTEGRAL EQUATIONS WITH TWO NONLINEARITIES

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We consider a system of integral equations on the positive semiaxis with two monotone nonlinearities. With various particular representations of matrix kernels and nonlinearities, this system arises in many branches of mathematical physics. A constructive existence theorem for a non-negative, non-trivial and bounded solution is proved. We also study the asymptotic behavior of the solution at infinity. Under additional restrictions on the nonlinearities and matrix kernels, a uniqueness theorem for a solution, in a certain class of bounded vector functions, is proved. At the end, specific examples of matrix kernels and nonlinearities are given.

Keywords: matrix kernel, nonlinearity, bounded solution, monotonicity, convergence, limit relation.

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